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Invariant Representation of Images under Change in Illumination

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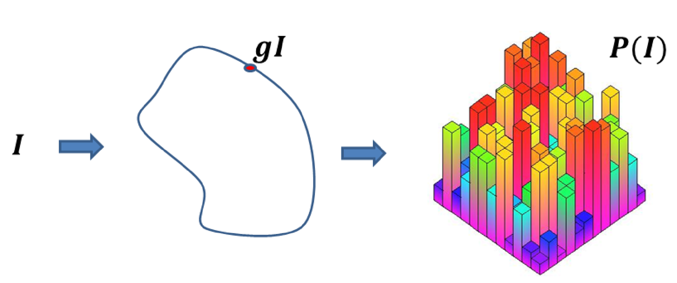
Introduction

∗ It has been shown [1] that an image representation based on a neurobiological model of simple-complex cells (Hubel and Wiesel model) is selective and invariant to group transformations and reduces the sample complexity of a learning task.
∗ This project aims to construct an invariant image representation under changes in illumination.
∗ We tested for illumination on two data sets: the SUFR data set [3] containing synthetic faces under 7 illumination conditions and the extended Yale Face Database B [4] containing images of human subjects under 64 illumination conditions.

Theoretical framework

G: finite compact group. *Equivalence relation* between images: $I \sim I' \Leftrightarrow \exists g_i \in G,\ I = g_i I'$. For an image I we have the image orbit, $O_I = \{g_1 I, \cdots, g_M I\}$, which is uniquely associated to a probability distribution P_I:

Theorem 1 $I \sim I' \Leftrightarrow O_I \sim O_{I'} \Leftrightarrow P_I = P_{I'}$



Implementing invariance and discriminability

∗ **Cramer-World Theorem**: probability distributions are uniquely determined by all of their one dimensional projections

$$P_I \equiv P_{I'} \Leftrightarrow P_{\langle I,t \rangle} \equiv P_{\langle I',t \rangle}$$

∗ **Group average**: Let G a finite compact group. The *group average* of any function $f : R \to R$ is

$$\bar{f}(x) \equiv \sum_i f(g_i x)$$

and we have $\bar{f}(x) = \bar{f}(g_i x),\ \forall g_i \in G$.

An invariant and selective signature

The following approximates the distribution of the values $\langle g_i I, t^k \rangle$ for one template t^k:

$$\mu_n^k(I) = \sum_i \left(\langle g_i I, t^k \rangle \right)^n$$

$\forall I, I'$ images we have:

Theorem 2 *The signature* $\mu(I) = (\mu_1^1(I), \cdots, \mu_N^K(I))$

- *is* **invariant** *i.e.* $\mu(g_i I) = \mu(I),\ \forall g_i \in G$
- *is* **selective (among classes)** *i.e.* $\mu(I) = \mu(I')\ \ iff\ \ I \sim I'$

If G is unitary, we have $\langle g_i I, t^k \rangle = \langle I, g_i^{-1} t^k \rangle \Rightarrow$ we have only one orbit of an arbitrary template to implement invariance of an image seen only once.

Framework for illumination transformations

∗ **Contrast Functions**: continuous, monotonic, and positive functions acting on an image.

We assume the contrast function $e : \mathbb{R}^2 \times \mathbb{R} \to \mathbb{R}^+$ has the following properties:

1. $e(x, 0) = 1$
2. $e(x, \zeta) e(x, \zeta') = e(x, \zeta \circ \zeta'),\quad \zeta, \zeta' \in \mathbb{R}_+$

where $\circ$ is a group composition. The transformation of an image I is:

$$I_\zeta(x) \equiv I(x, \zeta) = e(x, \zeta) I(x, 0).$$

and similarly for template t. We have the following signature for an image under change in illumination:

$$\mu_n^k(I) = \int_0^\infty \left(\langle I_0, t_\zeta^k \rangle \right)^n d\zeta$$

Illumination for the case of face images

For each of the datasets, we let I_0 be a face under one illumination condition and I_ζ be all other illumination conditions of the face. Our set of templates $\{t_\zeta^k\}$, contained faces under all illumination conditions with $t^k \neq I$.

Illumination for the case of Mondrian images

Let $e(x, m) = e^{mx}$ and $m = [-0.01, 0.01]$. For the illumination transformation,

$$e(x, m) I(x) = e^{mx} I(x) \equiv I_m(x),$$

and by theorems 1 and 2 we have the following invariant signature:

$$\mu_n^k(I) = \int_0^\infty \left(\langle I_0, t_m^k \rangle \right)^n dm$$

Methods

∗ **Invariance**: We calculated the euclidean distance between images and their transformations. For invariance we tested:

$$||\mu(I_0) - \mu(I_\zeta)||^2 \sim 0$$

where we fixed 1 template, with $\mu(I) \in \mathbb{R}^{n \times k}$, and averaged the euclidean distance over images and their transformations.

∗ **Selectivity**: We calculated the euclidean difference between different images. For selectivity we tested:

$$||\mu(I_0^i) - \mu(I_0^j)||^2 \nsim 0,\ \ i \neq j$$

where we averaged over many templates.

∗ **Control**: We defined a "fake orbit" as a set of randomly selected images from the datasets.

Results

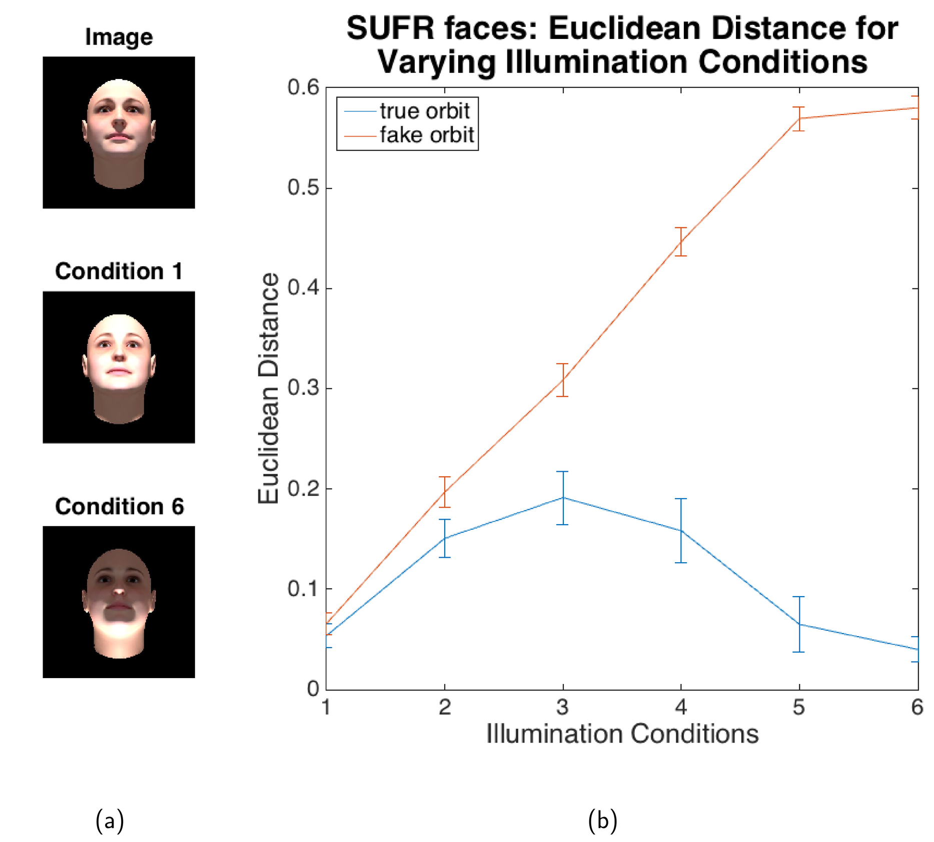


Figure 1: (a) Example of a face from SUFR dataset and two illumination conditions. (b) Euclidean distances between faces within the same orbit for the true and fake orbits.

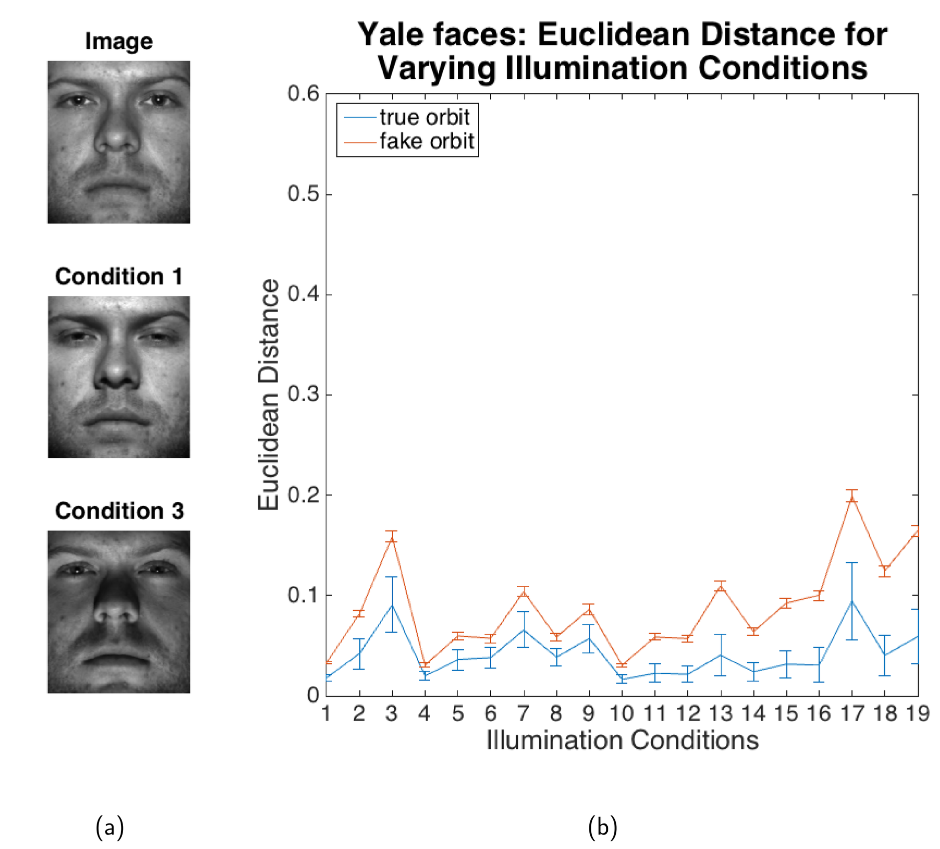


Figure 2: (a) Example of a face from YaleFace dataset and two illumination conditions. (b) Euclidean distances between faces within the same orbit for the true and fake orbits.

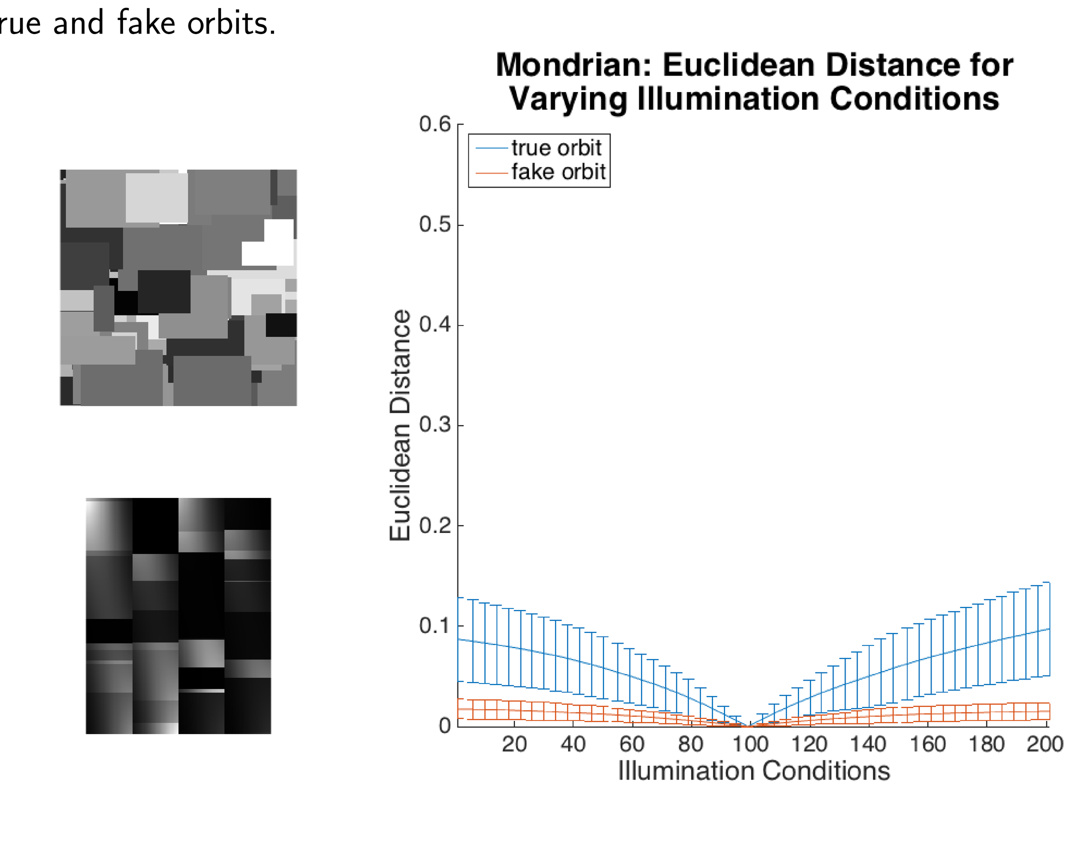


Figure 3: (a) Example of a Mondrian image (top) and strips of the Mondrian under different illumination conditions (bottom). (b) Euclidean distance between images within the same orbit for the true and fake orbits.

Discussion

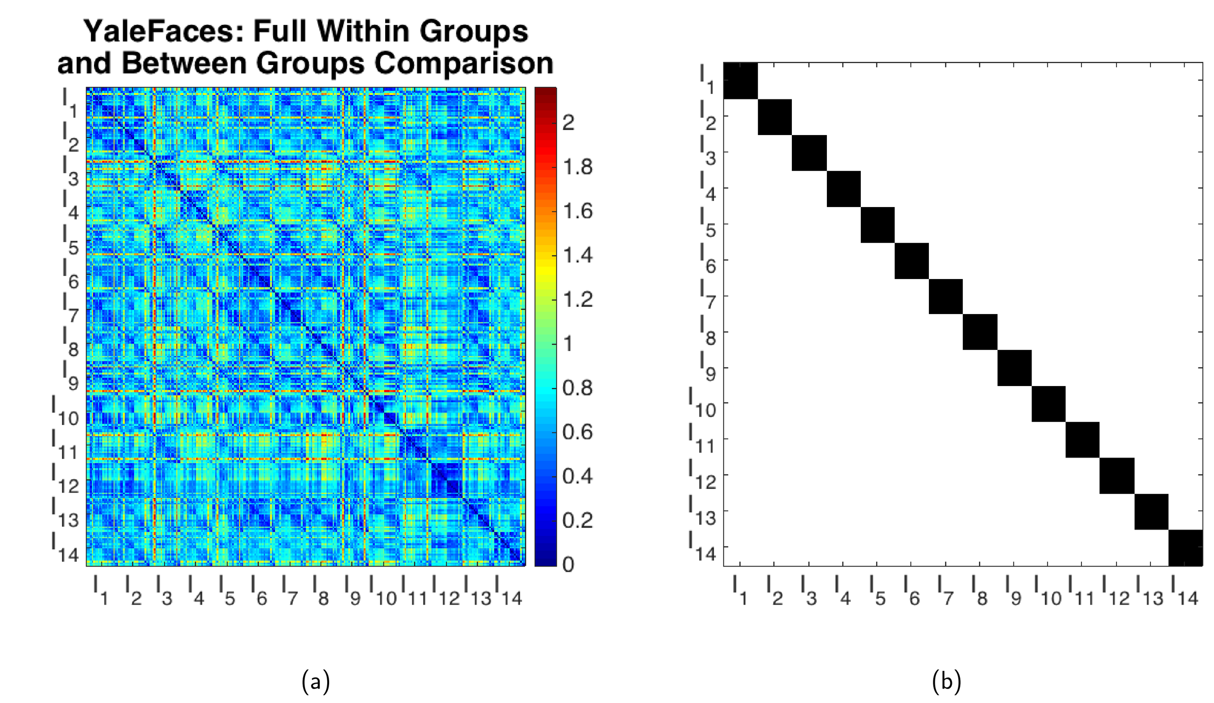


Figure 4: (a) Within groups and between groups comparison of faces. Within groups measured the difference between faces and each of their illuminations (I and each $g_i I$). Between groups measured the difference between different faces (I_i and I_j). (b) The ground truth distance matrix. Each block is the within groups comparison, and white is the between groups comparison.

∗ **Invariance**: Results suggest improved invariance compared to the control for face images but not for Mondrian images.
∗ **Selectivity**: Results were inconclusive (see Figure 4).

∗ **Possible explanations**:

1. normalization of signatures/other distance metrics
2. limited orbit size (number of transformed images)
3. assumptions made about illumination transformations/models

Summary

We considered a framework for invariance under illumination transformations of images and tested the theory on real and artificial data. For face images with a limited number of orbit samples, we observed invariance but not selectivity. Future work will include finer illumination models and experiments with more controlled, synthetic light conditions.

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References

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